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TECHNICAL REPORT ON THE WISCONSIN VALUE-ADDED MODEL:

ACADEMIC YEAR 2024-25

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INTRODUCTION

This report describes the value-added growth model and score calculation methodology used by Education Analytics to estimate the contribution of Wisconsin public schools to growth in student knowledge. The 2024-25 model uses assessment data from the Wisconsin Forward Exam, PreACT Secure, and the ACT.

The report is divided into three sections. The first section describes the data sets used to produce the value-added scores. The second section describes the model used to calculate value-added scores for schools in Wisconsin. The third section presents some properties of the value-added results.

Conceptually, value-added analysis is the use of statistical techniques to isolate the component of student knowledge gains that is attributable to schools. Other factors that may influence student knowledge gains include prior knowledge and student characteristics. In practice, value-added growth models focus on the improvement students make on assessments from one year to the next. Value-added growth models often consider (control for) differences in student characteristics, such as economic disadvantage and disability, to help isolate the impact of schooling.

The model used in Wisconsin controls for the available set of student characteristics to identify the extent to which schools contribute to the improvement of student achievement outcomes. Once school-level value-added scores are calculated, these are averaged to obtain district scores. To calculate the final value-added scores, up to three years of results are combined: 2022-23, 2023-24, and 2024-25.

ANALYSIS DATA SETS

Before estimation can take place, a substantial amount of work is required to assemble the data sets used to produce the value-added estimates. A separate data set is prepared for each grade and subject. In total, 16 data sets are produced, covering grades 4 through 11 for English language arts (ELA) and math.

Each analysis data set includes students who have: (1) a test score in 2024-25 (the posttest) in the grade and subject being considered; (2) test scores in 2023-24 in both ELA and math (the pretests) in a single grade below their current grade; and (3) full academic year (FAY) status in their school or district in the 2024-25 school year.

The model also includes students in private schools participating in voucher school programs (referred to as Private School Choice Programs in Wisconsin). All voucher recipients are included in modeling. In addition, some private schools receiving voucher funds opt to receive

a value-added score that includes all attending students, including those students not receiving vouchers.

Student-level variables

POSTTEST AND PRETEST SCORES

The value-added model uses test scores from the 2022-23, 2023-24, and 2024-25 administrations of the Forward Exam, the PreACT Secure, and the ACT. The Forward Exam was administered to students in grades 3 through 8; the PreACT Secure to students in grades 9 and 10; and the ACT to students in grade 11. The model produces school-level estimates for grades 4 through 11 in ELA and math based on performance on the 2024-25 assessment. The ELA model uses the 2024-25 ELA score as the posttest. The math model uses the 2024-25 math score as the posttest. Both models include pretests in both ELA and math, both from the year before the posttest in 2023-24 and, when available, from two years before the posttest in 2022-23.

GENDER, RACE/ETHNICITY, ECONOMIC DISADVANTAGE, AND MIGRANCY

Gender, race/ethnicity, economic disadvantage, and migrancy information are drawn from the Wisconsin Information System for Education data (WISEdata) elements submitted by schools and districts. More specifically, the values for these variables are drawn from the Spring Demographic Snapshot of WISEdata captured on May 24, 2025.[1] In the data set used for value-added modeling, students are assigned the gender, race/ethnicity, economically disadvantaged status, and migrant status reported in the post-test year. Gender categories are male, female, and non-binary. Race categories are American Indian/Alaskan Native, Asian, Native Hawaiian/Pacific Islander, Black/African American, Hispanic/Latino, White, and multi-racial. The model uses an indicator for economic disadvantage and an indicator for whether students are migrant.

ENGLISH LANGUAGE PROFICIENCY CLASSIFICATION

Eight indicators for English-language proficiency (ELP) are included in the dataset. Students with ELP classifications of 1 through 5 are English-language learners in ascending levels of proficiency. Students with an ELP classification of 6 are those who were formerly classified as

[1] WISEdata is a dynamic data delivery system. Snapshots capture a static version of the data as it was delivered to Wisconsin DPI on a given date. The Spring Demographic Snapshot taken near the end of the school year was for the purpose of supplying demographic characteristics to associate with student assessment results.

English learners. Students with an ELP classification of 7 are those who were never English Learners. ELP classification is drawn from the WISEdata Snapshot mentioned above. An eighth indicator is created for students for whom ELP status is unknown.

DISABILITY

The dataset includes five indicators for students with disabilities according to their primary disability code. There are separate indicators for emotional/behavioral disability (EBD), learning or intellectual disability (LD/ID), autism (A), and speech/language disability (SL). All other disability codes are grouped into a single indicator for other disabilities. For public schools, disability status is based on a student having an active individualized education program (IEP) or individualized service plan (ISP) between December 1 and June 30 of the 2024-25 school year. For private Choice schools, disability status is based on data submitted at the time of the May 24, 2025 Spring Demographic Snapshot.

School enrollment

The dataset includes indicators of full academic year (FAY) status at the school and district level for 2024-25. For the Wisconsin accountability systems and therefore for the Wisconsin value-added model, FAY is defined as being enrolled from the third Friday of September through completion of statewide testing. Students who are FAY at the school level are included in school-level value-added scores. Some students are FAY in a district but not in a single school because of mobility within the district. These students are included in the district value-added scores but not in the school value-added scores.

Students attending private school

The data set also includes test scores for students participating in one of the Private School Choice programs in Wisconsin. These students receive a voucher to attend private school. All participating private schools receive a value-added score based only on students in Choice programs (i.e., those receiving vouchers).

In addition, participating private schools are given the option to receive a second report card and value-added score that includes all students in the school. Such schools are denoted as “opt-in” schools because they opted to receive the second non-compulsory score. In order to calculate value-added scores for “opt-in” schools, students attending “opt-in” schools but not using vouchers are added to the original data set. This new expanded data set includes all public-school students, and students receiving vouchers, and all other students attending “opt-in” schools. Then, value-added scores for “opt-in” schools are computed by re-estimating the value-added model using the expanded data set. This value-added score is

based on all students in the school – those receiving vouchers as well as those not receiving vouchers.

Descriptive statistics of analysis samples

Tables 1 and 2 describe the sample used for the 2024-25 school year. Note that the sample includes students from public schools and private schools participating in one of the Choice programs in Wisconsin. The private school students include students attending schools that opted in to receive a score for all their students regardless of whether an individual student is participating in Choice.

Table 1. Math Sample

Variable	Grade 4	Grade 5	Grade 6	Grade 7	Grade 8	Grade 9	Grade 10	Grade 11
Number of Students	54,978	55,396	55,379	55,704	55,476	54,404	57,030	56,949
Number of Public School Students	51,437	51,883	51,749	52,103	52,034	51,538	53,808	53,940
Number of Students in Choice Programs	2,752	2,728	2,793	2,738	2,524	2,330	2,493	2,260
Number of Private School Students not in Choice Programs	545	526	524	506	499	191	412	398
Total Number of Private School Students	3,297	3,254	3,317	3,244	3,023	2,521	2,905	2,658
Number of Public Schools	1,061	1,001	690	651	660	536	545	562
Number of Private Schools	181	184	176	170	170	75	78	76
Number of Public School District Codes	431	431	431	430	431	391	388	390
Posttest Mean Score	1580.871	1603.748	1622.942	1641.015	1661.493	17.069	18.139	19.331
Posttest Standard Deviation	50.176	49.183	46.161	54.273	57.838	3.721	4.556	5.663
Math Pretest Mean Score	1551.819	1576.133	1601.545	1618.576	1634.686	1656.763	17.132	18.198
Math Pretest Standard Deviation	53.410	55.617	50.741	50.157	60.913	59.137	3.841	4.675
ELA Pretest Mean Score	1561.643	1583.006	1601.048	1616.787	1627.378	1644.927	16.208	17.492

Variable	Grade 4	Grade 5	Grade 6	Grade 7	Grade 8	Grade 9	Grade 10	Grade 11
ELA Pretest Standard Deviation	52.913	52.452	49.787	49.295	55.666	60.008	5.280	5.965
Proportion in ELP Level 1	0.012	0.007	0.006	0.007	0.007	0.007	0.005	0.006
Proportion in ELP Level 2	0.020	0.012	0.009	0.016	0.015	0.012	0.008	0.008
Proportion in ELP Level 3	0.035	0.031	0.024	0.034	0.032	0.027	0.026	0.024
Proportion in ELP Level 4	0.013	0.022	0.020	0.010	0.012	0.014	0.012	0.011
Proportion in ELP Level 5	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001
Proportion in ELP Level 6 (former English learners)	0.016	0.030	0.045	0.048	0.047	0.053	0.063	0.062
Proportion in ELP Level 7 (not English learners)	0.891	0.886	0.883	0.874	0.875	0.877	0.877	0.880
Proportion ELP Unknown	0.013	0.012	0.012	0.011	0.011	0.009	0.009	0.008
Proportion Female	0.488	0.488	0.491	0.487	0.486	0.486	0.486	0.489
Proportion Male	0.512	0.512	0.508	0.513	0.513	0.513	0.512	0.510
Proportion Non-Binary	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001	0.001	0.001
Proportion Asian	0.044	0.047	0.046	0.046	0.043	0.043	0.043	0.041
Proportion African American	0.095	0.095	0.090	0.086	0.088	0.081	0.072	0.066
Proportion Hispanic	0.144	0.144	0.145	0.148	0.148	0.145	0.140	0.139
Proportion Native American	0.009	0.009	0.009	0.009	0.009	0.008	0.008	0.008
Proportion Native Hawaiian or Other Pacific Islander	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001
Proportion Two or More Races	0.059	0.056	0.055	0.054	0.050	0.048	0.046	0.045
Proportion Special Education: Emotional Behavioral	0.008	0.009	0.009	0.011	0.011	0.010	0.010	0.009
Proportion Special Education: Learning/ Intellectual	0.051	0.059	0.058	0.054	0.050	0.047	0.042	0.037

Variable	Grade 4	Grade 5	Grade 6	Grade 7	Grade 8	Grade 9	Grade 10	Grade 11
Proportion Special Education Autism	0.018	0.018	0.016	0.016	0.016	0.014	0.013	0.012
Proportion Special Education: Speech/ Language	0.037	0.025	0.013	0.008	0.005	0.002	0.002	0.002
Proportion Special Education: Other	0.045	0.042	0.044	0.043	0.044	0.042	0.040	0.034
Proportion Economically Disadvantaged	0.439	0.434	0.423	0.415	0.403	0.388	0.360	0.344
Proportion Migrant	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001

Table 2. English Language Arts (ELA) Sample

Variable	Grade 4	Grade 5	Grade 6	Grade 7	Grade 8	Grade 9	Grade 10	Grade 11
Number of Students	54,997	55,422	55,401	55,760	55,533	54,278	56,930	55,823
Number of Public School Students	51,451	51,904	51,758	52,151	52,079	51,419	53,715	52,869
Number of Students in Choice Programs	2,759	2,731	2,799	2,744	2,528	2,327	2,492	2,232
Number of Private School Students not in Choice Programs	544	526	524	506	501	191	412	398
Total Number of Private School Students	3,303	3,257	3,323	3,250	3,029	2,518	2,904	2,630
Number of Public Schools	1,062	1,001	690	652	662	536	545	562
Number of Private Schools	182	184	176	170	170	75	78	75
Number of Public School District Codes	431	431	431	430	431	391	388	390
Posttest Mean Score	1582.917	1600.009	1617.587	1628.949	1647.130	16.251	17.492	18.890
Posttest Standard Deviation	53.901	49.524	49.795	54.261	62.094	5.450	6.165	5.766
ELA Pretest Mean Score	1561.621	1582.994	1601.039	1616.755	1627.348	1645.058	16.213	17.595
ELA Pretest Standard Deviation	52.925	52.453	49.806	49.317	55.673	59.961	5.281	5.958

Variable	Grade 4	Grade 5	Grade 6	Grade 7	Grade 8	Grade 9	Grade 10	Grade 11
Math Pretest Mean Score	1551.787	1576.115	1601.536	1618.55	1634.637	1656.916	17.136	18.262
Math Pretest Standard Deviation	53.445	55.616	50.746	50.160	60.954	59.044	3.842	4.681
Proportion in ELP Level 1	0.012	0.007	0.006	0.007	0.007	0.007	0.005	0.004
Proportion in ELP Level 2	0.020	0.012	0.009	0.016	0.015	0.012	0.008	0.008
Proportion in ELP Level 3	0.035	0.031	0.024	0.034	0.032	0.027	0.026	0.024
Proportion in ELP Level 4	0.013	0.022	0.020	0.010	0.012	0.014	0.012	0.011
Proportion in ELP Level 5	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001
Proportion in ELP Level 6 (former English learners)	0.016	0.030	0.045	0.048	0.047	0.053	0.063	0.063
Proportion in ELP Level 7 (not English learners)	0.891	0.886	0.883	0.874	0.875	0.877	0.877	0.882
Proportion ELP Unknown	0.013	0.012	0.013	0.011	0.011	0.009	0.009	0.008
Proportion Female	0.488	0.488	0.491	0.487	0.486	0.486	0.486	0.494
Proportion Male	0.512	0.512	0.508	0.513	0.513	0.513	0.512	0.505
Proportion Non-Binary	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001	0.001	0.001
Proportion Asian	0.044	0.047	0.046	0.046	0.043	0.043	0.043	0.041
Proportion African American	0.095	0.095	0.091	0.086	0.088	0.080	0.071	0.065
Proportion Hispanic	0.144	0.144	0.145	0.148	0.148	0.145	0.140	0.137
Proportion Native American	0.009	0.009	0.009	0.009	0.009	0.008	0.008	0.008
Proportion Native Hawaiian or Other Pacific Islander	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001
Proportion Two or More Races	0.059	0.056	0.055	0.054	0.050	0.048	0.046	0.044
Proportion Special Education: Emotional Behavioral	0.008	0.009	0.009	0.011	0.011	0.010	0.010	0.008

Variable	Grade 4	Grade 5	Grade 6	Grade 7	Grade 8	Grade 9	Grade 10	Grade 11
Proportion Special Education: Learning/ Intellectual	0.051	0.059	0.058	0.054	0.050	0.047	0.042	0.035
Proportion Special Education: Autism	0.018	0.018	0.016	0.016	0.016	0.014	0.013	0.011
Proportion Special Education: Speech/Language	0.037	0.025	0.014	0.008	0.005	0.003	0.002	0.002
Proportion Special Education: Other	0.045	0.042	0.044	0.043	0.044	0.042	0.04	0.032
Proportion Economically Disadvantaged	0.439	0.434	0.423	0.415	0.403	0.387	0.359	0.339
Proportion Migrant	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001

Transformation of assessment variables

All test scores are transformed to a rank-based z-statistic scale with means equal to zero and standard deviations equal to one in each grade and subject. Thus, in the value-added model, all test scores were measured relative to the state means, and in units of the statewide standard deviations of test scores in given grades and subjects. The rank-based z-statistic transformation ranks scores and then assigns to them a z-statistic score (or z-score) based on the value associated with that rank in the standard normal distribution.

The transformation to the rank-based z-statistic scale is made for several reasons. Normalizing the test scores makes it easier to interpret the coefficients in the value-added model, given that they are now measured in units of the standard deviation across students in scores on the posttest assessment.

In addition, using a rank-based normalization reshapes the distribution of test scores to be the same (standard normal) for all grades, subjects, and assessments. This reshaping makes the model more robust to changes in assessments, such as the transition from the Forward assessments to the PreACT Secure. It typically reduces the extent of nonlinearity in the relationships between the posttest and pretests, particularly when the posttest and pretests are different assessments.

This transformation is extended to the conditional standard errors of measurement (CSEMs) of the pretests, which are used to account for pretest measurement error when estimating the value-added regression (see the “Value-added regression” section below). Transforming the CSEMs involves multiple steps, implemented separately for each scale score value. First, we

repeatedly simulate measurement error around the scale score, creating a set of repeated, simulated scale scores with measurement error. These simulated scores are drawn from a normal distribution with a mean at the value of the original scale score and a standard deviation at the CSEM associated with the original scale score. Next, we transform the simulated scale scores using the same transformation as that which was used to transform the original scale scores. Last, we compute the standard deviation of the transformed simulated scores. This computes the CSEM on the rank-based z-statistic scale. There is no need to extend this transformation to the CSEM of the posttest. This is because posttest measurement error simply becomes incorporated into the error term of the regression and does not need to be accounted for when estimating the value-added regression.

Pretest score reliabilities

The reliabilities for pretest scores are displayed in Table 3. The reliability of an assessment is the extent to which differences in test scores reflect true differences in student knowledge rather than just random variation. Across years, grades, and subjects, the reliability of the pretest scores on the rank-based z-statistic scale ranges from 0.891 to 0.937 on the Forward Exam, and from 0.875 to 0.900 on the PreACT Secure. These reliability values suggest that the vast majority of the variance of these test scores reflect actual differences in student knowledge of the content area.

Table 3. Pretest Reliabilities for the 2024-25 Model

Posttest Exam/Model	Posttest Grade	Test Subject	Pretest from 2022-23		Pretest from 2023-24	
			Assessment	Reliability	Assessment	Reliability
Forward	4	ELA	Forward	N/A	Forward	0.903
		Mathematics		N/A		0.931
	5	ELA	Forward	0.899	Forward	0.897
		Mathematics		0.925		0.937
	6	ELA	Forward	0.908	Forward	0.891
		Mathematics		0.929		0.925
	7	ELA	Forward	0.901	Forward	0.901
		Mathematics		0.920		0.931
	8	ELA	Forward	0.905	Forward	0.896
		Mathematics		0.929		0.926
PreACT Secure	9	ELA	Forward	0.901	Forward	0.895
		Mathematics		0.918		0.933
	10	ELA	Forward	0.910	PreACT Secure	0.900
		Mathematics		0.911		0.875
ACT	11	ELA	PreACT Secure	0.900	PreACT Secure	0.900
		Mathematics		0.875		0.875

When the pretest is the Forward assessment, the reliabilities in Table 3 are based on the transformed CSEMs described in the "Transformation of assessment variables" section. When the pretest is the PreACT Secure assessment, these reliabilities are the median reported in the grade 10 panel of Table 6.8 of the *PreACT Technical Manual* (ACT Education Corp., 2025). This is because using the transformed CSEMs for the PreACT Secure yielded unexpectedly low reliabilities and unexpected coefficients when the value-added regression was estimated. Consequently, an SEM equal to the square root of the difference between one and the reliability was used instead when the pretest was the PreACT Secure.

VALUE-ADDED MODEL

For the Wisconsin school-level model, 2024-25 value-added is estimated in mathematics and English language arts (ELA) using the Forward Exam in grades 4 through 8, the PreACT Secure in grades 9 and 10, and the ACT in grade 11. School single-year value-added scores reflect student growth from Spring 2024 to Spring 2025. Schools' value-added scores are averaged to obtain a district-level value-added score, using the number of students attributed to each school as weights.[2] The single-year value-added scores for 2024-25 are averaged with the two most recent prior value-added scores (2023-24 and 2022-23) to produce a multi-year average score that smooths year-to-year variability in value-added scores.

The model, in brief

The value-added growth model is defined by six equations: a "best linear predictor" value-added model defined in terms of true student posttest and pretest achievement (i.e., student achievement in the absence of test measurement error) and five measurement error models for observed posttest and pretest scores:

Student achievement: $y_{3i} = \zeta + \lambda_2 y_{2i} + \lambda_2^{alt} y_{2i}^{alt} + \lambda_1 y_{1i} + \lambda_1^{alt} y_{1i}^{alt} + \beta' X_i + \alpha' S_i + e_i$ (1)

Posttest measurement error: $Y_{3i} = y_{3i} + v_{3i}$ (2)

Same-subject, once-lagged pretest measurement error: $Y_{2i} = y_{2i} + v_{2i}$ (3)

Other-subject, once-lagged pretest measurement error: $Y_{2i}^{alt} = y_{2i}^{alt} + v_{2i}^{alt}$ (4)

Same-subject, twice-lagged pretest measurement error: $Y_{1i} = y_{1i} + v_{1i}$ (5)

Other-subject, twice-lagged pretest measurement error: $Y_{1i}^{alt} = y_{1i}^{alt} + v_{1i}^{alt}$ (6)

where:

- the subscript i denotes each individual student;

[2] Note that students who changed schools within a given district within a year are included in the district's score but not in a school score (see "School enrollment" section).

- y_{3i} is true posttest achievement;
- y_{2i} and y_{2i}^{alt} are true pretest achievement, one year before posttest achievement, in the same subject and in the other subject (math in the ELA model, ELA in the math model), with slope parameters λ_2 and λ_2^{alt};
- y_{1i} and y_{1i}^{alt} are true pretest achievement, two years before posttest achievement, in the same subject and in the other subject (math in the ELA model, ELA in the math model), with slope parameters λ_1 and λ_1^{alt};
- X_i is a vector of characteristics of student i, with slope parameter vector β;
- S_i is a vector of indicators for school;
- α is a vector of school effects;
- e_i is the error in predicting posttest achievement given the explanatory variables included in the model;
- Y_{3i} is observed posttest scores;
- v_{3i} is measurement error in posttest scores;
- Y_{2i} and Y_{2i}^{alt} are observed pretest scores, one year before the posttest, for the same subject and alternate subject, respectively;
- v_{2i} and v_{2i}^{alt} are measurement error in pretest scores, one year before the posttest, for the same subject and alternate subject, respectively;
- Y_{1i} and Y_{1i}^{alt} are observed pretest scores, two years before the posttest, for the same subject and alternate subject, respectively; and
- v_{1i} and v_{1i}^{alt} are measurement error in pretest scores, two years before the posttest, for the same subject and alternate subject, respectively.

Substituting the measurement error equations (2) through (6) into the student achievement equation (1) yields an equation defined in terms of student test scores:

$$\text{Observed test scores: } Y_{3i} = \zeta + \lambda_2 Y_{2i} + \lambda_2^{alt} Y_{2i}^{alt} + \lambda_1 Y_{1i} + \lambda_1^{alt} Y_{1i}^{alt} + \beta' X_i + \alpha' S_i + \varepsilon_i \quad (7)$$

where the error term ε_i includes both the original error component and the measurement error components:

$$\text{Error in observed test scores: } \varepsilon_i = e_i + v_{3i} - \lambda_2 v_{2i} - \lambda_2^{alt} v_{2i}^{alt} - \lambda_1 v_{1i} - \lambda_1^{alt} v_{1i}^{alt} \quad (8)$$

Estimating the observed test scores equation (7) without controlling for pretest measurement error yields biased estimates of all parameters, including the value-added effects. This bias stems from the fact that measurement error in pretest scores causes the error term (8), which includes the measurement error components v_{2i}, v_{2i}^{alt}, v_{1i}, and v_{1i}^{alt}, to be correlated with observed pretest scores. The desired parameters, as defined in equation (1), can be estimated consistently if external information is available on the variance of measurement error for the pretests; approaches for consistent estimation in the presence of measurement error are

described in detail in Fuller (1987). Information about the variance of test measurement error is obtained from the conditional standard errors of measurement (CSEMs) provided alongside the test scores or from reliabilities published in the assessment technical report.

In contrast to measurement error in the pretest score variables, measurement error in the posttest does not cause any distortions in commonly used regression approaches and can safely be overlooked. This is because we do not expect posttest measurement error v_{3i} to be correlated with observed pretest scores or any of the other right-hand-side variables in the regression equation (7). We do not expect any such correlation because there is no reason to think that a student's good or bad luck on the posttest administration should have anything to do with their past test scores, their demographic characteristics, or their school assignment. Given the absence of such a correlation, the presence of posttest measurement error v_{3i} in the regression error term in (8) will not bias coefficient estimates if it is overlooked. In fact, from the perspective of estimation technique, we can think of posttest measurement error v_{3i} as operating no differently from the structural error e_i.

Value-added regression

As mentioned, the value-added growth model is estimated using a least-squares regression approach that corrects for measurement error in the pretest score variables. It estimates the coefficients λ, β, and α by regressing posttest scores on the pretest scores, student characteristic variables, and a full set of school fixed effects. This regression is estimated using an approach that accounts for measurement error in the pretests Y_{2i}, Y_{2i}^{alt}, Y_{1i}, and Y_{1i}^{alt}. Recall from equation (8) above that v_{2i}, v_{2i}^{alt}, v_{1i}, and v_{1i}^{alt}, the measurement error components of the pretests, are part of the error term ε_i. As a result, estimating the regression using ordinary least squares (without controlling for pretest measurement error) will lead to biased estimates. The regression approach employed accounts for measurement error by removing the variance in the pretests that is attributable to measurement error. To illustrate the measurement error corrected regression, re-cast the above value-added regression equation into matrix form:

$$Y_t = Y_{t\text{-}\ell}\lambda + W\delta + \varepsilon$$

where Y_t is an N × 1 matrix of post-test scores, $Y_{t\text{-}\ell}$ is an N × 4 matrix of same-subject and other-subject pre-test scores Y_{2i}, Y_{2i}^{alt}, Y_{1i}, and Y_{1i}^{alt}; λ is a 4 × 1 matrix made up of λ_2, λ_2^{alt}, λ_1, and λ_1^{alt}; W is an N × K matrix of the X demographic variables and S school indicators, δ is a K × 1 matrix of the β and α coefficients, and ε is an N × 1 matrix of error terms. The biased ordinary-least-squares estimates of the coefficients in λ and δ are equal to:

$$\begin{bmatrix}\hat{\lambda}_{OLS}\\ \hat{\delta}_{OLS}\end{bmatrix} = \begin{bmatrix}Y'_{t-\ell}Y_{t-\ell} & Y'_{t-\ell}W\\ W'Y_{t-\ell} & W'W\end{bmatrix}^{-1}\begin{bmatrix}Y'_{t-\ell}Y_t\\ W'Y_t\end{bmatrix}$$

The measurement-error-corrected estimates of the coefficients in λ and δ are equal to:

$$\begin{bmatrix}\hat{\lambda}_{EIV}\\ \hat{\delta}_{EIV}\end{bmatrix} = \begin{bmatrix} Y'_{t-\ell}Y_{t-\ell} - \left(\frac{N-K-4}{N}\right)\sum_{i=1}^{N} V_{it-\ell} & Y'_{t-\ell}W \\ W'Y_{t-\ell} & W'W \end{bmatrix}^{-1} \begin{bmatrix} Y'_{t-\ell}Y_t \\ W'Y_t \end{bmatrix}$$

where $V_{it\text{-}\ell}$ is a 4×4 variance-covariance matrix of the errors of measurement of the variables in $Y_{t\text{-}\ell}$ for student i. This model is described in section 2.2 of Fuller (1987).

The variables in the model

In addition to posttest and pretest scores, the student characteristic variables included in the value-added model (the X variables in equation 1) include gender, race/ethnicity, ELP category (indicators reflecting each of ELP levels 1-5, former ELP, not ELP, and ELP unknown), an indicator for economic disadvantage, disability status (indicators for emotional/behavioral, learning/intellectual, autism, speech/language, and all others), and a migrant status indicator. No higher order terms or interactions of terms are used in the model. Refer to the section "Student-level variables" for a more complete description of the categories that make up each student characteristic variable.

Incorporating students with only two years of scores

The estimation approach above produces school value-added results based on the growth of students with test scores in all three years (2022-23, 2023-24, and 2024-25). To include students with test scores in 2024-25 and 2023-24 but not in 2022-23, we estimate a value-added model that is identical to that described above except that it does not include the pretest variables y_{1i} and y_{1i}^{alt}. We then produce, for each student, a growth residual equal to an estimate of $\alpha' S_i + \varepsilon_i$, using the coefficients from the complete model that includes y_{1i} and y_{1i}^{alt} when the pretest scores Y_{1i} and Y_{1i}^{alt} are available, and using the coefficients from the model that does not include y_{1i} and y_{1i}^{alt} when the pretest scores Y_{1i} and Y_{1i}^{alt} are not available. This growth residual is demeaned by subtracting its mean across students by grade and subject and regressed on a full set of school indicators S_i using ordinary least squares. This produces school value-added results for each school by grade and subject.

Frequency of lowest observed scale scores

In some grades, a disproportionate number of students received Forward Exam math scores at the lowest observable scale score (LOSS). We present the proportion of students with scores at the LOSS in Table 4. The substantive number of students at the LOSS was a primary reason for converting scale scores using the rank-based z-statistic transformation for use in the value-added model. This conversion sets scores at the LOSS (and all other levels) to values

corresponding to a normal distribution of student achievement across the state, which keeps scores at the LOSS from entering the model as outliers that may distort the value-added regression.

Table 4. Percentage of Students at Test Floor (Lowest Observable Scale Score, LOSS) for Pre- and Posttests

	Posttest Grade	Test Subject	Percent at Posttest Floor	Percent at Math Pretest Floor	Percent at ELA Pretest Floor
Included in Growth Analysis Data Set	4	ELA	<0.1	1.0	<0.1
		Mathematics	<0.1	1.0	<0.1
	5	ELA	<0.1	1.9	<0.1
		Mathematics	<0.1	1.9	<0.1
	6	ELA	<0.1	0.5	<0.1
		Mathematics	<0.1	0.5	<0.1
	7	ELA	<0.1	1.3	<0.1
		Mathematics	<0.1	1.3	<0.1
	8	ELA	<0.1	1.9	<0.1
		Mathematics	<0.1	1.9	<0.1
	9	ELA	<0.1	0.7	<0.1
		Mathematics	<0.1	0.7	<0.1
	10	ELA	<0.1	<0.1	<0.1
		Mathematics	<0.1	<0.1	<0.1
	11	ELA	<0.1	<0.1	<0.1
		Mathematics	<0.1	<0.1	<0.1

Aggregation to multiple-grade value-added

The value-added regression to obtain school value-added results is performed separately for each grade and subject combination. For schools that have results for more than one grade level, these estimates are averaged across grades, using the number of students attributed to the school and grade as weights, to produce multiple-grade value-added results. Before aggregation, value-added results are normalized by subject and grade, so they are on a similar scale (i.e. with a mean of 0 and a standard deviation of 1). This normalization is done by dividing the results by a sampling-error-adjusted estimate of the standard deviation of within-grade value-added. This aggregation is made separately at the elementary/middle (grades 4-8) and high school (grades 9-11) levels.

Shrinkage of value-added

At all levels (school, district, student group), the value-added results are 'shrunk' to obtain value-added scores, using an Empirical Bayes multivariate shrinkage technique described in Longford (1999). This procedure brings value-added scores based on smaller sample sizes closer to the state average, so that schools with fewer students are not overrepresented among the highest- and lowest-value-added score cases simply due to randomness. It also reduces year-by-year variation in value-added scores for schools with small student populations.

This multivariate shrinkage approach begins with single-year value-added results for the 2024-25 and 2023-24 school years. Let $\hat{\alpha}_{kt}$ be the value-added result for school *k* in year *t*. We can group the value-added results for a given school *k* into a *T* x 1 column vector $\hat{\alpha}_k$, where *T* is the number of years in which value-added is estimated for school *k*. (In this application, *T* will usually be 2, although it will equal 1 in schools in which value-added is estimated in 2024-25 but not 2023-2024 or vice versa.) Also let α_{kt} be the true value-added (which is unmeasured, and equal to what the value-added result would be in the absence of sampling error) for school *k* in year *t*, which can be grouped by school into a *T* x 1 column vector α_k. Let the variance of α_k be the *T* x *T* matrix $Var[\alpha_k] = \Omega$, which reflects the within-year variance and across-year covariance of true value-added across schools. Also let the variance of $\hat{\alpha}_k$ conditional on α_k be the *T* x *T* matrix $Var[\hat{\alpha}_k|\alpha_k] = \Sigma_{kk}$, which reflects the within-year variance and across-year covariance of sampling error in $\hat{\alpha}_k$.

We produce shrunk value-added scores using the following equation:

$$\alpha_k^* = \Omega[\Omega + \Sigma_{kk}]^{-1}\hat{\alpha}_k$$

where α_k^* is a *T* x 1 column vector of shrunk value-added scores for school *k* over the *T* years in which value-added is measured for school *k*. The shrunk value-added scores α_k^* are single-year scores that do not overrepresent small schools among the highest- and lowest-value-added score cases and also exhibit less year-to-year variability than before shrinkage. These scores are later aggregated across years into a weighted three-year average score (see the "Multi-Year Aggregation" section below).

The expected mean squared error of the shrunk value-added scores α_k^* is equal to:

$$EMSE_k = \Omega - \Omega[\Omega + \Sigma_{kk}]^{-1}\Omega$$

In practice, we use estimates of Ω and Σ_{kk} to estimate α_k^* and its expected mean squared error. The estimate of the matrix Σ_{kk} is the estimated variance-covariance matrix of the value-added results in $\hat{\alpha}_k$. Let $\hat{\sigma}_{t\tau kk}$ be the entry of this matrix in the row corresponding to $\hat{\alpha}_{kt}$ and the column corresponding to $\hat{\alpha}_{k\tau}$. The diagonal entries of this matrix are the squares of the estimated standard errors of the value-added results in $\hat{\alpha}_k$.

The diagonal entries of Ω, which are equal to the variance of α_{kt} across schools in a given year *t* and which we denote ω_{tt}, are estimated by computing the variance across schools *k* within year *t* of the value-added results $\hat{\alpha}_{kt}$, then subtracting from that the average across schools *k* within year *t* of $\hat{\sigma}_{ttkk}$, the estimated squared standard error of $\hat{\alpha}_{kt}$. This estimates the variance of the true school value-added for each year *t*, excluding variance due to randomness in the value-added results. The square root of this variance measure is also used for normalizing value-added results by grade before aggregation to school-level value-added results across all grades. The off-diagonal entries of Ω, which we denote $\omega_{t\tau}$ and are equal to the covariance of α_{kt} and $\alpha_{k\tau}$ across schools between years *t* and τ, is estimated by computing the covariance of the value-added results $\hat{\alpha}_{kt}$ and $\hat{\alpha}_{k\tau}$, and then subtracting from that the average error covariance estimate $\hat{\sigma}_{t\tau kk}$.

Student group value-added

Value-added is also estimated for student groups defined by certain student characteristics. Specifically, we calculate value-added scores for:

- the seven race/ethnicity groups;
- students with and without disabilities;
- economically disadvantaged and non-economically disadvantaged students;
- English-language learners[3] and non-English-language learners;
- students who were proficient (and not proficient) in the same subject in the previous year; and
- students who are in (and not in) a target group made up of students who were the lowest scoring 25% within their school in the same subject in the previous year.

To produce the student group value-added scores by school for all groups other than those defined by proficiency or by target group membership, we first estimate school-level value-added results for each student group. These are estimated by computing the sum of the school effects and the residual, $\alpha' S_i + \varepsilon_i$, for each student, and then averaging this sum across students for each student group in the school. We then shrink each of these results for 2024-25 jointly with the schools' results for that student group in 2023-24 using a multivariate shrinkage approach that considers correlations in school- and student group-level value-added across groups and years. This produces single-year student group results that exhibit less year-to-year variability and reduce the extent to which across-subgroup variability reflects random variation. After shrinkage, the student group results are re-centered for consistency so that the average of school value-added scores across the school's student groups (weighted by

[3] The English-language learners group for purposes of student group value-added estimation includes students who reached English language proficiency in the last four years.

the number of students in each group) is equal to the school's value-added score for all students. These are later aggregated across years into a weighted three-year average student group score (see the “Multi-Year Aggregation” section below).

Value-added by proficiency status

To produce proficient and not proficient student group results by school, we regress the sum of the school effects and residual, $\alpha' S_i + \varepsilon_i$, on same-subject rank-based z-scores from the year immediately prior within each school. This regression is estimated in a way that accounts for measurement error in pretest scores, using approaches described in section 2.5 of Fuller (1987); this is a modification of the approach used to estimate the main value-added regression (described in the “Value-added regression” section above) that is better suited to the smaller, within-school samples. This regression produces a separate intercept and slope for each school. The intercept estimates the school's effect on a student with the average rank-based z-score for that subject. The slope estimates the relationship between student pretest scores and test score gains at that school.

Next, over the sample of schools, we regress the slopes on a quintic of average same-subject prior-year rank-based z-score by school, and use the residual from this regression as the new slope. The effect of this adjustment is to make the slopes very neutral to average prior achievement by school. The adjustment is made to reduce the possibility that differences in slope across schools are a result of differences across schools in the magnitude of pretest measurement error, which is substantially associated with prior achievement. It also reduces the possibility that differences in slope emerge from nonlinearities in the relationship between posttest and pretest. The intercept for each school is also adjusted so that the estimated school effect at the average prior score within the school is the same before and after the adjustment. This adjustment is implemented starting in 2024-25.

We shrink these intercepts and slopes for 2024-25 jointly with intercepts and slopes for 2023-24 using a multivariate shrinkage approach that considers correlations of the intercepts and slopes both with each other and over time. This produces shrunk, single-year intercepts and slopes that exhibit less random variability, including from year to year. After shrinkage, the intercepts are re-centered to achieve consistency between school value-added scores for all students and proficient/not-proficient student group value-added scores. Re-centering sets value-added for a student with the average rank-based z-score at the school equal to the school's all-students value-added score for that year. School intercepts and slopes (after shrinkage and re-centering) are then used to produce value-added scores for each year for a representative non-proficient student and a representative proficient student. The representative non-proficient student is defined as one with a pretest rank-based z-score of

-0.67, which corresponds to the average z-score for non-proficient students across grades and subjects statewide in 2017-18. The representative proficient student is defined as one with a pretest rank-based z-score of +0.86, which corresponds to the average z-score for proficient students across grades and subjects statewide in 2017-18. The 2017-18 averages, which precede the use of rank-based z-scores in the value-added model and were based on linear z-scores, continue to be used so that the interpretation of the proficient and not proficient student group results remained consistent over time. The single-year value-added scores for proficient and not proficient students are aggregated into three-year weighted average value-added scores for proficient and not proficient students (see the "Multi-Year Aggregation" section below).

Value-added for the target group

To produce target group value-added scores by school, we do not simply apply the approach used for the other categorical student groups such as English-language learner or students with disabilities. Doing so would produce scores that are biased upward in the target group (which is lower-scoring) and downward in the non-target group (which is higher-scoring). This is because the pretest scores used to assign students to the target group are inevitably measured with some degree of error. Due to this pretest measurement error, some students will have been erroneously assigned to the target group because their pretest scores are lower than what their true achievement would indicate. Since pretest measurement error does not affect posttest scores, these students are likely, in the absence of any adjustment, to have higher value-added estimates than their true knowledge gains would indicate. Similarly, because of pretest measurement error, some students will have been erroneously assigned to the non-target group. These students are likely, in the absence of any adjustment, to have lower value-added estimates than their true knowledge gains would indicate.

In 2021-22, we approached this bias by initially producing value-added results in the same way that we did for the other categorical student groups but then making an adjustment before shrinkage that subtracted from the value-added results an estimate of the bias. Beginning in 2022-23, we approached this bias in a new way that produced growth residuals that could be averaged across students to produce target group value-added results without further adjustment if desired. This was implemented by re-estimating the value-added regression with the observed pretests replaced by predictions of true pretest achievement and with the school fixed effects replaced by interactions between the school fixed effects and an indicator for target group status. The regression is estimated by two-stage least squares, with the predictions of true pretest achievement instrumented with the observed pretest scores. We describe these approaches in the Appendix.

After adjusting for bias using either approach, we subtracted, from each school's target group value-added result, the average of target group value-added results across all schools.

Likewise, we subtracted, from each school's non-target group, the average of non-target group value-added results across all schools. This is because we were concerned, even after adjustments for pretest measurement error, that the average difference between the target and non-target group measures statewide did not necessarily reflect a statewide difference in school effectiveness between the two groups. In particular, we were concerned that it could reflect a possible non-linearity in the relationship between the posttest and the pretests. It is possible that, even in the absence of a target group being identified, the relationship between the posttest and the pretest may "bend" in a way that growth appears to become faster or slower statewide among lower-achieving students. This could appear to be a statewide effect of target group membership, even if it is only a side effect of the shape of the posttest-pretest relationship under "business as usual" conditions. Subtracting out the statewide mean of the target and non-target group value-added results eliminated this statewide difference and avoided the possibility of attribution of a difference that may not be causal.

Beginning in 2024-25, an additional adjustment was made, implemented before the step described in the previous paragraph. This adjustment regresses the difference between target group and non-target group value-added on a quintic of average same-subject prior-year rank-based z-score by school. Target group and non-target group growth are then adjusted so that their difference is equal to the residual from this regression, while their average—weighted by the number of students in and not in the target group—is unchanged. The effect of this adjustment is to make the difference between target group and non-target group value-added very neutral to average prior achievement. This step was implemented to reduce the possibility that differences in value-added between students in the target group and students not in the target group were generated by differences in the magnitude of pretest measurement error across schools. It was also implemented to further reduce the possibility that these differences are generated by nonlinearities in the relationship between posttest and pretest.

The target and non-target group value-added results were shrunk using a bivariate shrinkage approach that accounts for the correlation of growth within schools between the target and non-target group. This step was implemented to reduce the impact of differences in growth between the target and non-target groups that were the result of random variability. This is the same approach as employed in shrinkage of value-added results for other student groups, such as disability, with the exception that the target group results were shrunk separately year by year rather than jointly across years. The exception was made so that a school's growth measure for the target group for a given year specifically reflected the growth of students in that target group in that year, in part to recognize schools that made substantial improvements from one year to the next in growth among students in the target group. The shrunk value-added scores were then re-centered within school to ensure that the average of school value-added scores across the target and non-target groups (weighted by the number of students in the two groups) aligns with the school's value-added score for all students. The value-added

scores for the target and non-target groups are later aggregated across years into a weighted three-year average score for each group (see the “Multi-Year Aggregation” section below).

We computed district-level results for the target and non-target groups by averaging the school-level results for each group across schools within the district. We did not include in district-level value-added results for the target and non-target groups students who were not enrolled in a single school for the full academic year. This is because the target group is defined by students' prior achievement level relative to other students within their school.

Final stage for estimation of school and district value-added scores

MULTI-YEAR AGGREGATION

Final school value-added scores are calculated as a weighted three-year average of scores for 2022-23, 2023-24, and 2024-25. The weights used are equal to the number of students included in the school's value-added score for a given year, multiplied by 1.5 for 2024-25, 1.25 for 2023-24, and 1.0 for 2022-23. The multi-year averaged value-added score includes the 2022-23 and/or 2023-24 value-added scores only if there are at least twenty students associated with that specific year's value-added scores. If only a single prior year value-added score enters into the average, then that prior year score is multiplied by 1. If no prior year scores enter the average, then the 2024-2025 score is multiplied by 1. All value-added scores, including the student group scores, are reported as a multi-year average using the weighting described above.

The multi-year average value-added scores are rescaled, based on the number of years included, to have a variance like that of a single-year value-added score. The rescaling is done because an average of value-added scores over multiple years will tend to have a lower variance across schools than a single-year value-added score, and the more years are included in the average, the lower the variance will tend to become. In the absence of rescaling, the highest and lowest value-added scores will be disproportionately among schools in which the average includes only one or two years rather than three. To implement the rescaling, we first compute the standard deviations of four different aggregations of overall value-added scores: a single-year score from 2024-25; a two-year weighted average from 2023-24 and 2024-25; a two-year weighted average from 2022-23 and 2024-25; and a three-year weighted average from 2022-23, 2023-24, and 2024-25. Then, we divide all of a given school's multi-year value-added scores by the standard deviation for the kind of aggregation of the school's overall value-added score, and then multiply it by the standard deviation for single-year overall scores from 2024-25. This rescaling puts the variances of the multi-year averages of overall value-added scores at approximately the same level, regardless of which years are included in the

average. It also makes the same adjustment to all of a given school's multi-year averages, whether they are of overall value-added or of student group value-added.

CALCULATING DISTRICT-LEVEL SCORES

Final district value-added scores are obtained by averaging the multi-year value-added scores for the schools in each district, with weights determined by the number of students included in each school's single-year value-added score for 2024-25. This same procedure is used for overall and student group value-added scores for the district. As mentioned earlier, the district scores include students if they were FAY at the district even if they were not FAY at any of the district's schools. Thus, students who moved from one school in a district to another school in the district are included. These students are incorporated into the district score using a placeholder school for each district for students who were FAY in the district but not FAY in any school in the district.

PROPERTIES OF THE VALUE-ADDED RESULTS

Coefficients on student-level variables in the model

The coefficients estimated in the value-added model are presented in Tables 5 and 6. To interpret these coefficients, note that both pretest and posttest scores are measured using rank-based z-scores; therefore, all coefficients are measured in the posttest standard deviation scale. For example, note that the coefficient on economic disadvantage is -0.038 in grade 5 math. This implies that students who are economically disadvantaged improved by about 0.038 standard deviations less on the grade 5 math test than otherwise similar students who are not economically disadvantaged.

It is important to keep in mind the standard errors of the coefficients when interpreting them. A span of 1.96 standard errors in both the positive and negative directions provides a 95 percent confidence range for a coefficient. Continuing with the example of the coefficient on economic disadvantage in grade 5 math, note that the standard error of this coefficient estimate is 0.005. This means that, while our best estimate of the difference in growth between students who are and are not economically disadvantaged is -0.038 standard deviations of 5th grade achievement, a 95 percent confidence interval for the difference ranges from -0.028 to -0.048 standard deviations.

Table 5. Coefficients on Student-Level Variables, 2024-25 Math

	Grade 4		Grade 5		Grade 6		Grade 7		Grade 8		Grade 9		Grade 10		Grade 11	
Variable	Coeff.	SE	Coeff.	SE	Coeff.	SE	Coeff.	SE	Coeff.	SE	Coeff.	SE	Coeff.	SE	Coeff.	SE
Math Pretest Score (lag 1)	0.788	0.006	0.650	0.012	0.462	0.011	0.745	0.013	0.590	0.014	0.383	0.015	0.419	0.009	0.590	0.011
ELA Pretest Score (lag 1)	0.076	0.006	0.034	0.011	0.147	0.011	0.073	0.012	0.106	0.012	0.104	0.014	0.131	0.008	0.058	0.010
Math Pretest Score (lag 2)	N/A	N/A	0.239	0.012	0.368	0.011	0.117	0.012	0.263	0.013	0.462	0.015	0.436	0.011	0.261	0.009
ELA Pretest Score (lag 2)	N/A	N/A	-0.010	0.011	-0.052	0.010	0.017	0.011	-0.047	0.011	-0.085	0.015	-0.071	0.009	0.027	0.009
ELP Level 1	0.085	0.067	0.068	0.069	0.018	0.057	-0.051	0.064	0.123	0.068	0.251	0.105	-0.024	0.134	-0.139	0.130
ELP Level 2	0.066	0.066	0.055	0.065	-0.013	0.052	-0.076	0.059	0.116	0.064	0.109	0.102	-0.048	0.130	-0.122	0.128
ELP Level 3	0.094	0.065	0.036	0.063	0.009	0.049	-0.086	0.057	0.066	0.062	-0.062	0.10	-0.103	0.127	-0.009	0.126
ELP Level 4	0.154	0.066	0.028	0.064	0.026	0.049	-0.052	0.060	0.043	0.064	-0.162	0.101	-0.151	0.129	0.009	0.127
ELP Level 5	0.020	0.250	0.016	0.120	0.071	0.094	-0.178	0.145	-0.362	0.159	-0.060	0.177	-0.004	0.225	-0.115	0.186
ELP Level 6	0.177	0.067	0.024	0.064	0.048	0.048	-0.067	0.058	0.019	0.063	-0.098	0.10	-0.122	0.127	0.010	0.126
ELP Level 7	0.111	0.065	0.008	0.063	0.031	0.048	-0.106	0.057	-0.005	0.062	-0.083	0.099	-0.131	0.127	-0.014	0.125
Female	-0.109	0.005	0.031	0.005	-0.057	0.004	-0.052	0.004	0.029	0.004	-0.067	0.006	-0.066	0.005	-0.072	0.005
Non-Binary	0.112	0.183	0.024	0.151	0.047	0.102	-0.187	0.080	0.119	0.089	0.105	0.086	-0.079	0.072	-0.139	0.065
Asian	0.048	0.012	0.087	0.011	0.026	0.011	0.018	0.011	0.054	0.012	0.024	0.015	-0.017	0.014	0.005	0.013
African American	-0.065	0.011	-0.010	0.010	-0.059	0.010	-0.019	0.010	-0.016	0.011	0.043	0.013	-0.031	0.013	-0.039	0.012
Hispanic	-0.029	0.008	-0.005	0.008	-0.025	0.007	-0.008	0.007	-0.029	0.008	-0.031	0.010	-0.019	0.010	-0.041	0.009
American Indian or Alaskan Native	-0.039	0.026	-0.031	0.024	-0.012	0.023	0.008	0.024	0.000	0.024	0.006	0.032	0.006	0.029	-0.035	0.027
Native Hawaiian or Other Pacific Islander	0.034	0.087	0.083	0.083	-0.074	0.067	-0.033	0.079	0.099	0.082	-0.305	0.117	-0.048	0.093	0.078	0.080
Two or More Races	-0.016	0.009	0.004	0.009	-0.033	0.009	-0.007	0.009	-0.025	0.010	-0.013	0.012	-0.015	0.012	0.010	0.011

	Grade 4		Grade 5		Grade 6		Grade 7		Grade 8		Grade 9		Grade 10		Grade 11	
Variable	Coeff.	SE	Coeff.	SE	Coeff.	SE	Coeff.	SE	Coeff.	SE	Coeff.	SE	Coeff.	SE	Coeff.	SE
Special Education EBD	-0.095	0.024	-0.140	0.021	-0.045	0.020	-0.053	0.019	-0.085	0.020	0.085	0.027	-0.045	0.025	-0.155	0.025
Special Education LD/ID	-0.066	0.010	-0.049	0.009	-0.048	0.009	-0.008	0.009	0.016	0.010	0.048	0.013	-0.032	0.013	-0.049	0.012
Special Education A	-0.181	0.016	0.003	0.016	-0.044	0.015	0.006	0.016	-0.003	0.017	0.038	0.022	-0.095	0.022	-0.103	0.020
Special Education SL	0.016	0.011	-0.007	0.013	0.014	0.016	-0.004	0.022	0.056	0.028	0.002	0.053	-0.060	0.062	-0.089	0.056
Special Education Other	-0.104	0.011	-0.062	0.010	-0.065	0.010	-0.020	0.010	0.011	0.010	0.038	0.013	-0.034	0.013	-0.099	0.013
Economic Disadvantage	-0.044	0.005	-0.038	0.005	-0.026	0.005	-0.019	0.005	-0.038	0.005	-0.033	0.006	-0.031	0.006	-0.050	0.005
Migrancy Status	0.162	0.198	-0.075	0.185	0.284	0.164	0.151	0.180	0.153	0.190	-0.174	0.238	-0.221	0.277	-0.221	0.229

Table 6. Coefficients on Student-Level Variables, 2024-25 ELA

	Grade 4		Grade 5		Grade 6		Grade 7		Grade 8		Grade 9		Grade 10		Grade 11	
Variable	Coeff.	SE	Coeff.	SE	Coeff.	SE	Coeff.	SE	Coeff.	SE	Coeff.	SE	Coeff.	SE	Coeff.	SE
Math Pretest Score (lag 1)	0.092	0.007	0.084	0.012	0.060	0.013	0.074	0.014	0.126	0.014	0.079	0.014	0.102	0.009	0.098	0.010
ELA Pretest Score (lag 1)	0.798	0.007	0.597	0.012	0.607	0.012	0.685	0.013	0.594	0.012	0.532	0.014	0.567	0.007	0.530	0.010
Math Pretest Score (lag 2)	N/A	N/A	0.010	0.013	0.003	0.013	-0.032	0.013	-0.003	0.014	-0.011	0.015	0.017	0.010	0.058	0.009
ELA Pretest Score (lag 2)	N/A	N/A	0.246	0.012	0.257	0.011	0.211	0.012	0.229	0.011	0.315	0.015	0.266	0.009	0.211	0.008
ELP Level 1	0.103	0.074	-0.078	0.074	-0.009	0.063	-0.039	0.070	0.028	0.071	0.488	0.103	0.277	0.127	-0.478	0.124
ELP Level 2	0.116	0.072	-0.104	0.070	0.046	0.058	-0.046	0.064	0.061	0.067	0.195	0.099	0.194	0.123	-0.216	0.120
ELP Level 3	0.133	0.072	-0.072	0.068	0.051	0.054	-0.021	0.063	0.061	0.065	0.034	0.097	0.237	0.121	0.000	0.118
ELP Level 4	0.177	0.073	-0.058	0.069	0.039	0.055	-0.037	0.066	0.053	0.067	-0.079	0.099	0.223	0.122	0.087	0.119
ELP Level 5	0.293	0.275	0.019	0.129	0.172	0.106	-0.260	0.159	0.039	0.165	0.386	0.173	0.173	0.213	0.263	0.178

	Grade 4		Grade 5		Grade 6		Grade 7		Grade 8		Grade 9		Grade 10		Grade 11	
Variable	Coeff.	SE	Coeff.	SE	Coeff.	SE	Coeff.	SE	Coeff.	SE	Coeff.	SE	Coeff.	SE	Coeff.	SE
ELP Level 6	0.219	0.074	-0.031	0.069	0.066	0.054	-0.019	0.063	0.034	0.065	-0.036	0.097	0.227	0.121	0.009	0.118
ELP Level 7	0.144	0.071	-0.063	0.068	0.025	0.053	-0.048	0.063	0.019	0.065	0.012	0.097	0.246	0.120	-0.011	0.117
Female	0.069	0.005	0.039	0.005	0.022	0.005	0.012	0.005	0.039	0.005	0.073	0.006	0.079	0.005	0.098	0.005
Non-Binary	-0.426	0.201	0.084	0.163	-0.011	0.115	0.007	0.088	0.169	0.093	0.291	0.083	0.155	0.069	0.027	0.061
Asian	0.048	0.013	-0.006	0.012	0.071	0.012	0.060	0.012	0.021	0.012	-0.012	0.015	0.020	0.014	0.022	0.013
African American	-0.032	0.012	-0.028	0.011	-0.010	0.011	0.003	0.011	0.017	0.011	-0.029	0.013	0.008	0.012	-0.037	0.011
Hispanic	0.017	0.009	0.001	0.008	0.000	0.008	0.003	0.008	0.025	0.008	-0.044	0.010	0.006	0.009	0.001	0.008
American Indian or Alaskan Native	0.005	0.029	-0.015	0.026	0.001	0.026	-0.007	0.026	0.050	0.025	0.000	0.031	-0.011	0.028	-0.034	0.026
Native Hawaiian or Other Pacific Islander	0.050	0.095	-0.029	0.089	-0.033	0.075	-0.068	0.086	0.170	0.086	-0.076	0.114	0.009	0.089	0.025	0.077
Two or More Races	-0.014	0.010	-0.013	0.010	-0.004	0.010	0.011	0.010	-0.004	0.010	-0.005	0.012	0.009	0.011	0.001	0.010
Special Education EBD	-0.070	0.026	-0.057	0.023	-0.011	0.023	-0.079	0.021	-0.080	0.021	0.213	0.026	0.075	0.024	-0.330	0.024
Special Education LD/ID	-0.060	0.011	-0.093	0.010	-0.078	0.010	-0.053	0.010	-0.047	0.010	0.189	0.013	0.048	0.012	-0.336	0.012
Special Education A	-0.111	0.018	-0.056	0.017	-0.090	0.017	0.023	0.017	-0.035	0.018	0.369	0.022	0.128	0.021	-0.305	0.020
Special Education SL	0.009	0.012	0.000	0.014	-0.011	0.018	0.039	0.024	0.032	0.030	0.084	0.052	0.038	0.058	-0.199	0.053
Special Education Other	-0.079	0.012	-0.075	0.011	-0.089	0.011	-0.038	0.011	-0.037	0.011	0.193	0.013	0.061	0.012	-0.310	0.012
Economic Disadvantage	-0.047	0.006	-0.018	0.005	-0.029	0.005	-0.008	0.005	-0.016	0.005	-0.010	0.006	-0.007	0.006	-0.057	0.005
Migrancy Status	0.227	0.217	0.173	0.199	0.084	0.183	0.130	0.198	0.169	0.198	-0.174	0.232	0.144	0.263	0.064	0.240

Test of model neutrality: Correlation with average prior attainment

In this test, we calculate correlations between value-added estimates and school-level prior attainment. The value-added estimates employed in these correlations are the multi-year averages of value-added, post-shrinkage, for all students for a given subject. School-level prior attainment is computed as the average of 2023-24 scale score in the same subject in the previous grade within the value-added regression data set. The scale scores are converted to (linear, non-rank-based) z-scores by subtracting out the scale score mean by subject and grade and dividing by the scale score standard deviation by subject and grade. This makes it possible to meaningfully average scale scores across grades when required. In the grade-level correlations, both value-added and prior attainment are measured by grade. In the overall correlation, both value-added and prior attainment are multi-grade measures.

This correlation is a method for validating whether the variables included on the right-hand side of our regression adequately control for school-level factors influencing value-added estimates. The higher the correlation magnitude, the higher the level of "non-neutrality". We do not necessarily expect these correlations to be zero, particularly if schools with students with higher prior attainment also tend to be more effective at facilitating growth among their students. However, we may be concerned if this correlation is especially high (particularly if greater than 0.5), since this may be a sign that the control variables are not sufficiently "leveling the playing field" between schools with students with lower and higher prior attainment.

Our results (displayed in Table 7) show a low correlation at the school-and-grade level and a modest correlation at the overall school level between average prior attainment—a measure of average performance in the previous year—and value-added. For example, the correlation between multi-year value-added in English language arts in grade 4 and average 2023-24 scale score in grade 3 in English language arts is 0.033, which is very low. For context, it is often difficult to see correlations with absolute values of less than 0.2 on a scatterplot. On the other hand, the correlation between multi-year, multi-grade value-added in math and average 2023-24 scale score in math is 0.313, which is more substantial, but not so large as to create concerns about the value-added measures. In general, schools were somewhat more likely to have a high value-added score than a low one if their students had high pretest scores rather than low pretest scores.

Table 7. Correlations between Prior Attainment and Value-Added

Subject	Grade 4	Grade 5	Grade 6	Grade 7	Grade 8	Grade 9	Grade 10	Grade 11	Overall
ELA	0.033	-0.014	0.080	-0.056	0.037	-0.074	0.245	0.071	0.154
Math	0.153	0.134	0.156	-0.015	0.255	-0.088	0.338	0.204	0.313

Correlation between Math and ELA value-added

Table 8 shows substantive positive correlations between the multi-year math and ELA value-added scores, post-shrinkage, within each school. Schools with high value-added scores in math also tended to have high value-added scores in ELA. This implies that schools with a higher-than-average impact in mathematics also had a higher-than-average impact in English language arts. This is not a surprising result, given that we would expect that many of the aspects of a school that facilitate high growth in one subject would also facilitate growth in other subjects.

Table 8. Correlations between Subjects

Subjects	Grade 4	Grade 5	Grade 6	Grade 7	Grade 8	Grade 9	Grade 10	Grade 11	Overall
2024-25 Math & ELA	0.621	0.585	0.650	0.492	0.366	0.684	0.480	0.569	0.585

CONTACT

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REFERENCES

ACT Education Corp. (2025), *PreACT Technical Manual*, August 2025, accessed at <https://www.act.org/content/dam/act/unsecured/documents/PreACT-Technical-Manual.pdf>.

Fuller, W. (1987). *Measurement Error Models*, John Wiley and Sons.

Longford, N. T. (1999). Multivariate shrinkage estimation of small area means and proportions. *Journal of the Royal Statistical Society* 162 (Part 2), 227-245.

APPENDIX: TARGET GROUP MODELS

As noted in the section "Value-added for the target group", producing the target group measures using the same approach as that used to produce other student group measures will lead to biases in the absence of any adjustment. We discuss in detail below the approaches used to eliminate this bias.

In 2021-22, we produced value-added results in the same way as other categorical student groups, but then, before applying shrinkage, made an adjustment that subtracted from the value-added results an estimate of the bias. This estimate of bias was based on the standard error of measurement of the pretest scores and an assumption that pretest score error is normally distributed. The adjustments were calculated as follows:

$$\text{Adjustment for target group: } adj_target_k = -\lambda \frac{\sigma^2_{v(k)}}{\sqrt{\sigma^2_{y*(k)}+\sigma^2_{v(k)}}} \frac{\phi(z_k)}{\Phi(z_k)} \quad \text{(A.1)}$$

$$\text{Adjustment for non-target group: } adj_nontarget_k = +\lambda \frac{\sigma^2_{v(k)}}{\sqrt{\sigma^2_{y*(k)}+\sigma^2_{v(k)}}} \frac{\phi(z_k)}{(1-\Phi(z_k))} \quad \text{(A.2)}$$

where adj_target_k and $adj_nontarget_k$ are the bias adjustments added to the target and non-target group results respectively for school *k;* λ is the coefficient on same-subject, once-lagged pretest; $\sigma^2_{y*(k)}$ is an estimate of the variance in school *k* of same-subject, once-lagged pretest adjusted for measurement error; $\sigma^2_{v(k)}$ is an estimate of the variance in school *k* of measurement error in the same-subject, once-lagged pretest; z_k is the cutoff score in school *k* for inclusion in the target group, after transforming to z-score; $\phi(.)$ is the standard normal probability density function; and $\Phi(.)$ is the standard normal cumulative distribution function.

Beginning in 2022-23 a different approach was used to deal with this bias. This approach employs predictions of the true pretest variables y_{2i}, y_{2i}^{alt}, y_{1i}, and y_{1i}^{alt} given all the right-hand-side variables in equation (7) in the "Value-added model" section, including pretest scores, student characteristic variables, and school indicator variables. The use of these predictions makes the adjustments employed in previous years unnecessary.

The predictions are produced using the following steps. First, let $Y_{t-\ell}$ be an N × 4 matrix of pretest scores Y_{2i}, Y_{2i}^{alt}, Y_{1i}, and Y_{1i}^{alt} — the row of $Y_{t-\ell}$ corresponding to student *i* is denoted as $Y_{it-\ell}$. Let *W* be an N × K matrix of the *X* student characteristic variables and *S* school indicator variables. The pretest scores $Y_{t-\ell}$ are regressed by ordinary least squares on the other variables *W*, producing a predicted component $\hat{Y}_{t-\ell}$ and a residual component $\hat{u}_{t-\ell}$.

$$\text{Auxiliary regression: } Y_{t-\ell} = W\pi + u_{t-\ell} = X\pi_1 + S\pi_2 + u_{t-\ell} \quad \text{(A.3)}$$

$$\text{Estimated predicted component: } \hat{Y}_{t-\ell} = W\hat{\pi} = X\hat{\pi}_1 + S\hat{\pi}_2 \quad \text{(A.4)}$$

$$\text{Estimated residual component: } \hat{u}_{t-\ell} = Y_{t-\ell} - \hat{Y}_{t-\ell} \quad \text{(A.5)}$$

The predicted component $\hat{Y}_{t-\ell}$ should not include pretest measurement error at all, given that we do not expect pretest measurement error to be predictable from other variables. However, the residual $\hat{u}_{t-\ell}$ has a substantial component that is pretest measurement error.

Second, we estimate the variance-covariance matrix of the component of the residual ($\hat{u}_{t-\ell}$) that is *not* pretest measurement error as follows:

$$\text{Estimated error-adjusted covariance: } \Omega_u = (1/N)\hat{u}_{t-\ell}'\hat{u}_{t-\ell} - (1/N)\sum_{i=1}^{N} V_{it-\ell} \qquad \text{(A.6)}$$

where $V_{it-\ell}$ is a 4 × 4 diagonal matrix with the squared conditional standard errors of measurement of the elements of $Y_{it-\ell}$ on the diagonal. The $(1/N)\hat{u}_{t-\ell}'\hat{u}_{t-\ell}$ term estimates the total variance of $\hat{u}_{t-\ell}$, while the $(1/N)\sum_{i=1}^{N} V_{it-\ell}$ term estimates the variance of the component of $\hat{u}_{t-\ell}$ that is pretest measurement error. Consequently, the difference between the two terms (Ω_u) estimates the variance of the component of $\hat{u}_{t-\ell}$ that is *not* pretest measurement error.

Third, we predict the non-measurement-error component of $\hat{u}_{it-\ell}$ as follows:

$$\text{Predicted non-error component: } \tilde{u}_{it-\ell} = \Omega_u\left(\Omega_u + (1/N_{k(i)})\sum_{i\in k(i)} V_{it-\ell}\right)^{-1}\hat{u}_{t-\ell}, \qquad \text{(A.7)}$$

where $N_{k(i)}$ is the number of students in the school attended by student *i* and $\sum_{i\in k(i)} V_{it-\ell}$ is the sum of $V_{it-\ell}$ over students in the school attended by student *i*. When the pretest is the Forward assessment, the prediction $\tilde{u}_{it-\ell}$ is an Empirical Bayes estimate that uses conditional standard errors of measurement (CSEMs) to account for the fact that the variance—and, consequently, the relevance—of pretest measurement error is likely to be greater in some schools than in others. When the pretest is the PreACT Secure, the estimate uses an SEM equal to the square root of the difference between 1 and the reliability of the assessment; this is because CSEMs implied an unexpectedly low reliability for the assessment and, consequently, using CSEMs yielded unexpected results.

Fourth, the prediction of the true pretests y_{2i}, y_{2i}^{alt}, y_{1i}, and y_{1i}^{alt}, which we denote as the vector $\tilde{y}_{it-\ell}$, is computed by adding $\hat{Y}_{it-\ell}$ to $\tilde{u}_{it-\ell}$.

$$\text{Predicted true pretest achievement: } \tilde{y}_{it-\ell} = \hat{Y}_{it-\ell} + \tilde{u}_{it-\ell} \qquad \text{(A.8)}$$

After producing these predicted true pretest scores $\tilde{y}_{it-\ell}$, we estimate an instrumental-variables regression in which the left-hand-side variable is the posttest score Y_{3i}; the right-hand-side variables are the predicted pretest scores $\tilde{y}_{it-\ell}$, the student characteristic variables X_i, and interactions between the school indicator variables S_i and an indicator for target group status; and the predicted pretest scores $\tilde{y}_{it-\ell}$ are instrumented with the pretest scores $Y_{it-\ell}$. This regression is estimated using two-stage least-squares. In the first stage, $\tilde{y}_{it-\ell}$ is regressed on $Y_{it-\ell}$, X_i, and the interactions between S_i and the target group indicator, producing a prediction $\hat{\tilde{y}}_{it-\ell}$. In the second stage, Y_{3i} is regressed on $\hat{\tilde{y}}_{it-\ell}$, X_i, and the interactions between S_i and the target group indicator.

$$\text{First-stage regression: } \tilde{y}_{it-\ell} = \tau_1'Y_{it-\ell} + \tau_2'X_i + \tau_3'(S_i \times target_i) + \eta_{it-\ell} \qquad \text{(A.9)}$$

$$\text{Prediction from first stage: } \hat{\tilde{y}}_{it-\ell} = \hat{\tau}_1'Y_{it-\ell} + \hat{\tau}_2'X_i + \hat{\tau}_3'(S_i \times target_i) \qquad \text{(A.10)}$$

Second-stage regression: $Y_{3i} = \lambda^{*\prime}\hat{\tilde{y}}_{it-\ell} + \beta^{*\prime}X_i + \alpha^{*\prime}(S_i \times target_i) + \epsilon_{it-\ell}$ (A.11)

We use the coefficients from this regression to create a special growth residual, equal to the posttest score Y_{3i} minus the products of $\hat{\tilde{y}}_{it-\ell}$ and X_i and their coefficient estimates in the second-stage regression.

Target group growth residual: $q_i{}^* = Y_{3i} - \hat{\lambda}^{*\prime}\hat{\tilde{y}}_{it-\ell} - \hat{\beta}^{*\prime}X_i$ (A.12)

These growth residuals can be averaged across students by school and target group status to produce target group value-added results at the school level that do not need to be further adjusted for pretest measurement error, although further adjustments may be made if the data suggest that they would be valuable.